

Practice Problems for Elementary Statistics

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Problem Set C

1. A random variable x has uniform distribution on the interval $[2, 9]$. Find the probability of each of the following events.

- | | |
|-------------------|-----------------|
| (a) $x < 3$ | (d) $x > 3.6$ |
| (b) $x = 5.5$ | (e) $1 < x < 7$ |
| (c) $3 < x < 5.5$ | (f) $x < 9$ |

2. A random variable x has uniform distribution on the interval $[3, 10]$. Find:

- | | |
|---------------------------|-----------------------------------|
| (a) $P(3.5 < x < 7.9)$ | (d) $P(x = 7.4)$ |
| (b) $P(5 \leq x \leq 12)$ | (e) $E(x)$ |
| (c) $P(x > 5)$ | (f) the standard deviation of x |

3. Several busses travel a certain route in Chicago so that a bus arrives at a bus stop on the route every 15 minutes. A tired shopper arrives at the bus stop at a random time in the afternoon. In this situation, it is reasonable to assume that the waiting time, x , in minutes, for the shopper until the bus arrives is uniformly distributed on the interval, $[0 \text{ min}, 15 \text{ min}]$. Find

- the probability the waiting time will be less than 5 minutes,
- the probability that x will be more than 12 minutes,
- the probability the waiting time will be less than 90 seconds.
- the mean and standard deviation of x .

4. Let z denote a random variable with standard normal distribution. Find $P(z < 1)$, $P(z \geq 2.54)$, and $P(-1.22 < z < 1.22)$.

5. For a standard normal random variable, z , give the following:

- | | |
|-----------------------------|----------------------------|
| (a) $P(0 \leq z \leq 1.23)$ | (f) $P(z < -1.23)$ |
| (b) $P(1 < z \leq 2)$ | (g) $P(z < -7.53)$ |
| (c) $P(z > 2.11)$ | (h) $P(-1.20 < z < 2.18)$ |
| (d) $P(z < 6.79)$ | (i) $P(0 \leq z \leq 8.2)$ |
| (e) $P(z > -2.31)$ | (j) $P(z = 0)$ |

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6. For a standard normally distributed random variable, z , find:
- | | |
|---------------------------------|--------------------|
| (a) $P(-1.75 \leq z \leq 1.75)$ | (d) $P(z > 1.645)$ |
| (b) $P(z < 2.32)$ | (e) $P(z = 2.12)$ |
| (c) $P(z > 1.96)$ | (f) $P(z > 0)$ |
7. Let z be a random variable with standard normal distribution. (a) Find a number, c , such that $P(z > c) = .025$. (b) Give the mean and variance of z .
8. Let z be a random variable with standard normal distribution. (a) Find a number c such that $P(z < c) = .80$. (b) Give the expected value and standard deviation of z .
9. Suppose x is a normally distributed random variable with mean 45 and standard deviation 6.7. Find
- | | |
|----------------------|----------------------|
| (a) $P(x < 30)$ | (d) $P(50 < x < 71)$ |
| (b) $P(45 < x < 55)$ | (e) $P(x = 48)$ |
| (c) $P(x < 58)$ | (f) $P(x > 45)$ |
10. A random variable, x , has normal distribution with mean 23 and standard deviation 3.5. Find the following probabilities.
- | | |
|-------------------------|-----------------------------|
| (a) $P(x < 32)$ | (d) $P(x = 28)$ |
| (b) $P(20 < x < 56)$ | (e) $P(x > 30)$ |
| (c) $P(15 \leq x < 21)$ | (f) $P(0 \leq x \leq 2.71)$ |
11. Given that x is normal with mean 55 and standard deviation 7.4, find
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|---|
| (a) $P(x > 70)$ |
| (b) $P(x > 45)$ |
| (c) a number c such that $P(x > c) = .95$ |
12. Suppose x is normal with mean 123 and standard deviation 9.4 . Find
- | |
|--|
| (a) $P(118 < x < 140)$ |
| (b) $P(x > 75)$ |
| (c) a number c such that $P(x > c) = .075$ |
13. At a given moment, the speeds of cars on a certain stretch of highway will be normally distributed with standard deviation 4.3 mph. It is found that 38% of the cars are traveling at speeds of 70 mph or faster. Find the mean speed of cars on this stretch of highway at any given moment.
14. A vending machine dispenses coffee into cups. The mean amount dispensed is 8 oz, and the amount dispensed is assumed to be normally distributed. The capacity of the cup into which the beverage is dispensed is 9 oz. It is observed that 6.5% of the time the cup overflows. Find the standard deviation of the amount of coffee dispensed into a cup.
15. A standardized test is given to students in a certain population. The resulting scores are normally distributed with mean 345 and standard deviation 28.2 . Find the 75-th and 90-th percentiles for the test scores.

16. A fair coin is tossed 120 times. Use a normal approximation to estimate the probability of fewer than 50 heads in the 120 tosses.
17. A certain coin has heads probability .6. The coin is tossed 12 times. (a) Find the probability of 7 heads in the 12 tosses. (b) Now the coin is tossed 200 times. Use a normal approximation with continuity correction to estimate $P(x = 120)$.
18. A fair die is tossed 144 times. On each toss the number the die shows is observed. Let x be the number of times in the 144 tosses that the die shows a 5. Use a normal approximation to estimate
- the probability that x exceeds 30
 - $P(20 < x < 32)$
 - $P(x \leq 33)$
19. A large population of adults is 58% women. (a) For a random sample of 9 people selected from this population, what is the probability the sample will contain exactly 3 women? (b) For a random sample of 90 people from this population, estimate the probability the sample will contain fewer than 40 women.
20. A large population of voters is believed to be 35% democrats and 25% republicans. A random sample of size $n = 350$ is selected from this population. Assuming the given percentages are correct, use a normal approximation with continuity correction to estimate
- the probability the sample will contain more than 90 republicans,
 - the probability the sample will contain at most 130 democrats.
21. Two fair dice are tossed and the sum of the numbers showing is observed. This experiment is done 100 times. Let x be the number of times in the 100 runs that the dice give a sum of 12. (a) Find $E(x)$. (b) Find σ_x . (c) Is the number of runs of the experiment large enough that a normal approximation would be appropriate for estimating $P(x \leq 7)$? Explain.
22. Here is a little project. Let x have binomial distribution with parameters n and p . For $p = 1/2$, what is the smallest value of n such that the normal approximation will be considered adequate for estimating the probability distribution of x . For that value of n , tabulate the exact cumulative distribution of x , the normal approximation with continuity correction, and the normal approximation without continuity correction. In the case of this value of n , for what integers k is the normal approximation to $P(x \leq k)$ more accurate without the continuity correction than with?
23. A random sample of size 2 is taken from a population of x -values, and the sequence of numbers in the sample is observed. The population distribution is,

x	0	3	5	7	8
$p(x)$.05	.05	.4	.3	.2

- Find the probability the sample is the sequence, 5, 7, in that order. (b) Find the probability the sample consists of the values 8 and 5 in either order. (c) Find the probability the sample mean is 5.

24. A random sample of size 3 is taken from a population of x -values whose distribution is,

x	2	3	5	6	7
$p(x)$.1	.1	.4	.3	.1

The sample is taken, one observation at a time, and the exact sequence of values is recorded.

- (a) Find the probability the sample is the sequence, 7, 6, 5, in that order.
- (b) Find the probability the values selected are 2, 6, and 5 in some order.
- (c) Find the probability the sample mean is 6.
- (d) Find the probability the sample mean is less than 2.4.

25. A population of x -values has the following distribution:

x	1	3	5
$p(x)$.4	.2	.4

Samples of size 2 are repeatedly selected at random from this population, and the sequence of numbers is recorded.

- (a) List all possible samples of size 2.
 - (b) Find the probability of each sample.
 - (c) Give the probability distribution of the sample mean in repeated sampling.
 - (d) Give the probability distribution of the sample variance in repeated sampling.
 - (e) Give the probability distribution of the sample median, M , in repeated sampling.
 - (f) Compute the mean, variance, and standard deviation of the random variables x , \bar{x} , and s^2 . Express your answers concisely in a little table.
26. A population consists of units on which a certain variable is measured. The variable can assume only the values 2, 4, and 8, with these three values occurring in the ratios 1:1:3, respectively.
- (a) Give the probability distribution of a random variable x that models this variable and population, and compute the population mean and variance.
 - (b) For random samples of size 2, give (i) the sampling distribution of the sample mean, and (ii) the sampling distribution of the sample variance.
 - (c) Find the expected value (mean) of the sample mean in repeated sampling for random samples of size two.
 - (d) Find the standard deviation of the sample mean in repeated sampling for random samples of size two. (This is also called the standard error of the mean.)
 - (e) Find the expected value of the sample variance for samples of size 2.

- (f) Find the standard deviation of the sample variance in repeated sampling for random samples of size two. (This is also called the standard error of the sample variance.)
27. A certain population consists of a very large number of units, each of which has a numerical value, x , associated with it. The only values of x that occur are 2 and 8. To be precise, $2/3$ of the units in the population have x -value equal to 2 and $1/3$ have x -value equal to 8.
- Compute the mean and variance of the random variable x over the whole population.
 - For random samples of size 3, give in tabular form, the sampling distributions of (i) the sample mean, and (ii) the sample variance.
 - Compute the expected value of the sample mean in repeated sampling, for samples of size 3, and explain what is meant when we say that the sample mean is an unbiased estimator of the population mean.
 - Give the standard error of the mean for samples of size 3.
 - Find $P(\bar{x} \geq 6)$ for samples of size 3.
 - Compute the expected value of the sample variance for samples of size 3, and explain what is meant when we say that the sample variance is an unbiased estimator of the population mean.
28. A random sample of size $n = 3$ is taken with replacement from a large heap of numbers. The numbers in the heap are 0's and 12's and no other number occurs. To be precise, $2/3$ of the numbers are 0's and $1/3$ are 12's.
- Compute the population mean and population standard deviation.
 - For random samples of size 3, give the sampling distribution of the sample mean, sample median, and sample standard deviation, and give the expected values of each of these random variables.
 - Compute the standard error of the sample mean and the standard error of the sample median for samples of size 3.
 - Is sample standard deviation an unbiased estimator of population standard deviation for this population? Explain.
29. A certain population consists of observational units on which a variable x is defined. This variable has only four possible values: 6, 7, 8, and 9. When a unit is selected at random from the population, the x -value for that unit is equally likely to be any one of the four possibilities. Thus the population is modeled by a random variable x whose possible values are 6, 7, 8, and 9, each with probability $1/4$. Treat the population as being infinitely large, so that when units are selected randomly, one after the other, the probability distribution of the x -value of the unit being selected is not influenced by which units have been selected earlier.
- Compute the mean and variance of x .
 - Suppose three numbers are drawn at random, one number at a time, from the population, and the sequence of numbers is observed. How many outcomes does the sample space for this experiment contain?

- (c) Give the standard error of the mean for samples of size 3.
30. A random sample from a population of numbers has $\bar{x} = 37.6$, $n = 25$, and $s = 2.98$. Find the (estimated) standard error of the mean.
31. A random variable x has three values, 0, 1, and 8, each equally likely. Let y be the random variable $4x + 6$.
- (a) Compute the mean, variance, and standard deviation of x .
- (b) Compute the mean, variance, and standard deviation of y .
- (c) For 64 independent random observations of the random variable x , give the expected value and standard error of the sample mean.
- (d) For 25 independent random observations of y , give the standard error of the sample mean.
32. The following list of numbers constitutes a random sample of x -values from a certain population.

2, 2, 5, 7, 9

- (a) Find the sample mean and sample standard deviation.
- (b) Give a point estimate of the standard error of the mean.
33. For the following list of data, estimate the standard error of the mean.

-3, -1, 0, 1, 1, 1, 4, 5

34. In this problem we will see what goes wrong when sample variance is computed with n in the denominator instead of $n - 1$. So let \hat{s}^2 be variance computed with n in the denominator. That is,

$$\hat{s}^2 = \frac{\sum(x - \bar{x})^2}{n}$$

As always, s^2 will denote sample variance computed with $n - 1$ in the denominator. Give the sampling distributions of \hat{s}^2 and s^2 for samples of size 3 from the population of x -values whose distribution is,

$$\begin{array}{c|cc} x & 0 & 6 \\ \hline p(x) & .5 & .5 \end{array}.$$

Which of the two statistics, \hat{s}^2 and s^2 , is an unbiased estimator of σ^2 in this example? Explain.

35. Is sample standard deviation an unbiased estimator of population standard deviation? Show that for samples of size 2 from the population of x -values whose distribution is,

$$\begin{array}{c|ccc} x & 1 & 3 & 9 \\ \hline p(x) & .4 & .3 & .3 \end{array},$$

sample standard deviation is a biased estimator of population standard deviation. As this example shows, s is usually a biased estimator of σ , in spite of the fact that s^2 is always an unbiased estimator of σ^2 .

36. A certain population of numbers has mean 28.7 and standard deviation 3.3. For samples of size $n = 35$, (a) give the standard error of the mean, and (b) use the central limit theorem to estimate $P(\bar{x} > 30)$.
37. A certain population of numbers has mean 70 and standard deviation 18. The population distribution is close enough to normal that the central limit theorem may be invoked for sample sizes as small as 6. A sample of size 9 is selected at random from this population. (a) give the standard error of the mean, and (b) use the central limit theorem to estimate the probability that the sample mean is between 60 and 80.
38. The heights of red pine trees in a certain population have $\mu = 46$ ft and $\sigma = 3.7$ ft. In addition, the heights have mound-shaped distribution, so that the sampling distribution of the sample mean will be essentially normal for samples of size 4 or more. Suppose a sample of size 12 is taken from this population and the mean height, \bar{x} , of the 12 trees in the sample is computed. (a) Find the standard error of the sample mean, and (b) estimate the probability that the sample mean is greater than 45 ft. and
39. A random sample of 49 observations is selected from a population of numbers with mean 87.5 and standard deviation 17.2. (a) Give the standard error of the sample mean, and (b) use the central limit theorem to estimate the probability that the sample mean is between 90 and 100.
40. A population of numbers has mean 45 and standard deviation 7.2. A sample of size $n = 32$ is drawn at random. (a) Give the standard deviation of the sample mean in repeated sampling, and (b) use the central limit theorem to estimate $P(\bar{x} \leq 50)$, $P(45 \leq \bar{x} < 42)$ and $P(42 \leq \bar{x} < 45)$.
41. A population of x -values has uniform distribution on the interval from 2 to 11. A sample of size 64 is drawn at random from this interval and the sample mean is computed. (a) Give the standard deviation of the sample mean in repeated sampling, and (b) use the central limit theorem to estimate $P(2.5 \leq \bar{x} \leq 7.5)$ and $P(x > 6.9)$.
42. A population of numbers has uniform distribution on the interval $[-1, 7]$. (a) For samples of size 4, give the standard error of the sample mean. (b) For a sample of size 36, give the standard error of the mean, and estimate $P(\bar{x} < 2)$, and $P(2 < \bar{x} < 2.1)$.
43. A certain population is normally distributed with $\mu = 75.8$ and $\sigma = 10.3$. A sample of size $n = 5$ is drawn at random from this population and the sample mean, \bar{x} , is computed. Find (a) $P(\bar{x} = 75.8)$, (b) $P(75.8 < \bar{x} < 76.0)$, (c) $P(80 \leq \bar{x} \leq 90)$, and (d) the standard error of the mean.
44. For a certain population of ripe tomatoes, the weight of a tomato chosen at random is approximately normal with mean 7.4 oz. and standard deviation .63 oz. These tomatoes are packed in boxes of 12 and shipped to grocery stores for retail sale. Since the tomato weights are approximately normal, the mean (and sum) of as few as 5

weights could safely be treated as having normal distribution. Let y be the total weight, in pounds, of the 12 tomatoes in a box. Find

- (a) μ_y
- (b) σ_y
- (c) $P(y < 5)$
- (d) $P(y \geq 5.58)$

45. From past records, it is known that the customers of a certain airline weigh on the average 146 lb, with standard deviation 23 lb. A certain flight of this airline has 88 passengers. Assume the 88 passengers constitute a random sample from the usual population of customers. Find the probability the combined weight of the 88 passengers exceeds 6.6 tons, and state any assumptions upon which your calculation depends.

46. A population of x -values has distribution,

x	2	4	5	7	9
$p(x)$.1	.1	.1	.2	.5

- (a) For a randomly selected value, x , from this population, find $P(x \geq 8)$.
- (b) For random samples of size 2, find $P(\bar{x} \geq 8)$
- (c) A sample of size 40 is drawn randomly from the population. Use the central limit theorem to estimate $P(\bar{x} \geq 8)$.

47. A population of observations of a quantitative variable, x , has distribution,

x	2	8
$p(x)$.3	.7

- (a) For a randomly selected value of x from this population, find $P(x \geq 6)$.
- (b) For a random sample of size 3 from this population, find $P(\bar{x} \geq 6)$
- (c) For a random sample of 35 observations from the population, estimate $P(\bar{x} \geq 6)$.
- (d) For a random sample of 1600 observations, estimate $P(\bar{x} \geq 6)$

48. A fair die is tossed 3 times and the mean, \bar{x} of the three resulting numbers is computed. Find (a) $P(\bar{x} = 1)$ (b) $P(\bar{x} = 4/3)$, and (c) $P(\bar{x} = 2)$.

49. A fair die is tossed 31 times and the average of all the numbers obtained is computed. Use the central limit theorem to estimate the probability this average is less than 2.

50. A die is tossed 64 times and the average of all the numbers obtained is computed. Assuming the die is fair, and using the central limit theorem, (a) estimate the probability the average is between 3 and 4, and (b) estimate the probability the sum of the 64 numbers obtained exceeds 250.

51. A certain variable x is defined for the units in a certain population. It is known that x has uniform distribution on the interval $[3, 12]$. A sample of size 81 is drawn randomly from the population, and the sample mean, \bar{x} is computed. Use the central limit theorem to estimate $P(\bar{x} \geq 10)$.
52. A sample of size $n=64$ is randomly selected from a population of adults. The mean weight of the adults in the sample is 157 lb with standard deviation 9.2 lb. Give a 90% confidence interval for the mean weight of people in the population.
53. A random sample of 40 red pine trees is identified in a certain large red pine plantation, and the heights of the 40 trees are measured. It is found that the mean height of the trees in the sample is 48 ft, with standard deviation 2.5 ft. (a) Give a 99% confidence interval for the mean height of the trees in the plantation. (b) Interpret your confidence interval.
54. A random sample of 64 measurements has $\bar{x} = 5.23$ and $s = .63$. Give
- (a) the estimated standard error of the mean,
 - (b) a 90% confidence interval for the population mean,
 - (c) a 95% confidence interval for the population mean, and
 - (d) a 98% confidence interval for the population mean.
55. A certain population is known to be normally distributed with variance, $\sigma^2 = 39.7$. A random sample of size $n = 10$ from this population has mean, $\bar{x} = 63.1$. Give a 90% confidence interval for the population mean, μ .
56. A random sample of size 9 is taken from a normally distributed population with standard deviation, 12.5. The mean of the sample is 105.6. Give a 95% confidence interval for the population mean. Keep four significant figure accuracy.
57. A random sample of size 91 is taken from a certain population of numerical measurements. The sample mean and sample standard deviation are 83.2 and 21.5 respectively. Give a 95% confidence interval for the mean of the population.
58. In past years, students in a certain professor's statistics class turned in the final exam after working on it for 1 hour 42 minutes on the average. This year, for the 74 students taking the same exam as in past years, the average time a student spent working on it was 1 hour 35 minutes with standard deviation 17 minutes. Assuming that the students this year are a random sample from the population of all students this year who might have taken the class, give a 90% confidence interval for the true mean working time for the population. With this assumption, may we conclude with 90% confidence that students this year at the university are generally faster than in past years?
59. In a random sample of 35 14-oz jars of XYZ-brand peanuts, the net weight of the peanuts in a jar averages 12.08 oz with variance .0289 oz². (a) Give a 95% confidence interval for the mean weight of peanuts per jar. (b) Interpret the confidence interval.
60. A physicist makes 50 measurements of a certain physical constant. The mean of the readings is 2.980, with sample variance .000529. Give a 90% confidence interval and a 95% confidence interval for the true value of the constant.

61. A fisherman catches 32 bluegills on Lake Onalaska one week. The total weight of the 32 fish is 13 lb 2 oz. It is assumed from past records that the standard deviation in weight in the population of catchable bluegills is 1.9 oz. Give a 95% confidence interval for the mean weight, in ounces, of bluegills in the population from which the fisherman is catching the fish, and list assumptions that are necessary for the confidence interval to be considered valid.
62. Nancy M., a first grade teacher, gives a standardized reading test in the month of May to her class of 14 students. In her class, the mean score is 129. According to records in the school district office, the mean and standard deviation of the scores of all first grade children in the school district when they take this test in May are 122 and 18.5 respectively. (a) Give a 95% confidence interval for the true mean score that Ms. M's teaching methods produce, assuming that her class of 14 is a random sample from the population of all first graders in the district, and assuming that exam scores are normally distributed. (b) With these assumptions, can we say with 95% confidence that Ms. M is a better than average reading teacher?
63. An high school student makes 31 measurements of the period of the star Delta Cephei. The mean of the measurements is 128.664 hours, with standard deviation, $s=.571$ hours. (a) Using the student's results, give a 95% confidence interval for the true period of Delta Cephei in days. Give your answer to four significant figure accuracy, and state any assumptions needed for the confidence interval to be valid for the true period of the star. (b) Find out the accepted period of Delta Cephei. Does this suggest that the student's measurements are biased? Explain. (c) Give the confidence interval in part (a) in days, hours, minutes and seconds, except only go down to the smallest unit of time measure that is less then the error term.
64. An exam has 90 questions. In grading these exams, each answer is marked correct or incorrect. The average percentage of correct answers made by 36 college students on this exam is 56% with standard deviation 18%. (a) Consider the 36 students to be a random sample from a certain target population, and give a 95% confidence interval for the mean percentage of correct answers that would be achieved by the students in this population. (b) Give a 95% confidence interval for the mean number of correct answers per exam that would be achieved by students in the target population, and interpret the interval.
65. A random sample from a certain normally distributed population of numbers consists of the following list of observations:

23, 28, 26, 19, 28, 25, 26

Give a 95% confidence interval for the population mean.

66. A population of numbers is normally distributed. A random sample of size $n = 4$ is taken, resulting in the sequence, 392, 413, 401, 406. Give a 90% confidence interval for the population mean.
67. It is found by a medical researcher that five patients who were treated with a new medication to reduce blood pressure, had their blood pressure reduced by an average

of $\bar{x} = 3.71\text{mm}$ with standard deviation $s = .34\text{mm}$. (a) Give a 95% confidence interval for the mean amount by which the medication will lower blood pressure in the population. (b) State any assumptions required for the validity of your confidence interval.

68. A random sample of size 14 from a normally distributed population has mean 3.2×10^{12} and standard deviation 4.6×10^{11} . Give a 95% confidence interval for the mean of the population.

69. A random sample of size 8 from a normally distributed population of numbers has sample mean and sample standard deviation 89.4 and 23.1 respectively. Give a 90% confidence interval for the population mean.

70. A sleepy student is almost always late to his 8:00 o'clock a.m. class, but is never absent. On several days selected more or less randomly, the teacher, who hates this student, discretely records the time this student walks in. The times recorded are,

8:05, 8:12, 8:01, 8:03, 8:21, 8:04, 7:56

Give a 90% confidence interval for the average number of minutes this student is late throughout the semester. (At least he does come.)

71. An emergency siren is tested every Monday morning. It is supposed to sound at exactly 9:00 a.m. However, the person who turns it on is not very precise. On four randomly chosen Mondays a concerned citizen records the times the siren sounds. The times are 9:02, 9:03, 9:03 and 9:03. Assume that sounding times are normally distributed. (a) Give a 95% confidence interval for the average number of minutes late the siren is every Monday. (b) Does it appear that we can say with 95% confidence that the siren sounds, on the average, after 9:00 a.m.? The very next Monday, there is additional evidence that the mean sounding time is after 9:00, for the siren sounds at 9:24 a.m., over 20 minutes late! (c) Using that time together with the previous four times, recompute the 95% confidence interval for the mean number of minutes the siren is late in sounding. (d) Isn't it strange that here evidence consistent with the hypothesis the siren tends to be late actually makes us less confident that that is generally so?

72. A random sample of size 2 from a certain normally distributed population consists of the data values 17 and 35. Give a 95% confidence interval for the population mean.

73. A sample of size 49 from a normally distributed population has mean 7.21×10^{-6} and standard deviation 4.6×10^{-7} . Give a 95% confidence interval for the mean of the population.

74. A sample of size 36 from a normally distributed population has mean 9.2×10^5 and standard deviation 5.6×10^4 . Give a 95% confidence interval for the mean of the population.

75. Twelve corn plants are grown under certain conditions and their heights are measured at maturity. For the 12 heights measured, the mean and standard deviation were $\bar{x} = 7.3\text{ft}$, and $s = 0.45\text{ft}$, respectively. Give a 98% confidence interval for the true mean height, in inches, of corn plants of this kind grown under the given conditions.

76. A professor teaches two statistics classes, one at noon and one at 1:00 p.m. One day he gives a test in both classes. Summary statistics for the exam scores in the two classes are as follows.

Class	enrollment	mean \pm std. dev
noon	36	53.1 ± 17.5
1:00	34	44.2 ± 11.3

- (a) Give a 95% confidence interval for the true mean exam score of each class. (b) Do the confidence intervals in part (a) overlap? Caution: even though the confidence intervals overlap it may still be true that the difference between the two sample means is statistically significant. This issue is treated in a later section where it is resolved with a confidence interval for $\mu_1 - \mu_2$ that takes into account the sampling distribution of $\bar{x}_1 - \bar{x}_2$.
77. A chemist makes 5 determinations of the boiling point of a certain chemical. The mean of his five measurements is 107.4 degrees centigrade with standard deviation 1.09 degrees. Give a 95% confidence interval for the true boiling point of the chemical and state any assumptions used.
78. For a random sample from a certain population, $\bar{x} = 3.05$, $s = .62$, and $n = 75$. Give a 95% confidence interval for the population mean and state any necessary assumptions.
79. A random sample from a certain population of numbers is as follows:

23, 23, 20, 27, 29, 28

- Give a 90% confidence interval for the mean of the population, and state any necessary assumptions.
80. Ten years ago the average weight of the third grade boys at a certain public school was 48 lb. In a study of 24 third grade boys this year, the mean weight was 51.2 lb. and the standard deviation was 9.7 lb. Does this give statistically significant evidence that the third grade boys today weigh more than third grade boys ten years ago? Explain in one or more full sentence, using a 95% confidence interval.
81. A random sample from a population of adults consists of 67 men and 43 women. Give a 95% confidence interval for the proportion of men in the population.
82. A random sample of 60 income tax returns contains 19 with serious errors. Give a 95% confidence interval for the proportion of returns with serious errors on them in the population.
83. A random sample of 245 deer in the state of Wisconsin contains 140 does and 105 bucks. Give a 95% confidence interval for the proportion of does in the deer population in Wisconsin.

84. A random survey of 100 registered voters in a certain county reveals that 27 of these are democrats, 18 are republicans, 12 belong to some other political party and the rest are independent. (a) Give a 99% confidence interval for the proportion of registered voters who are democrats. (b) Give a 95% confidence interval for the proportion of registered voters who are independents.
85. A random sample from a certain population of voters consists of 45 republicans and 120 non-republicans. (a) Give a 90% confidence interval for the proportion of republicans in the population. (b) Interpret this confidence interval.
86. A random sample from a certain population of adults consists of 31 who smoke and 124 who do not smoke. (a) Give a 98% confidence interval for the proportion of smokers in the population. (b) Give a 98% confidence interval for the percentage of smokers in the population.
87. On January 30, 2003, a random sample of 1231 California voters contained 220 who supported Arnold Schwarzenegger for Governor. Give a 95% confidence interval for the true proportion of California voters who supported Schwarzenegger at that time, and interpret the interval.
88. A random sample of size $n=100$ is taken from the population of fourth grade students in the public school system of a certain city. It is determined that 35% of the students in the sample are overweight, and of these, half are sufficiently overweight to be classified as obese. (a) Give a 95% confidence interval for the percentage of fourth grade students in the population who are overweight, and interpret the interval. (b) Give a 90% confidence interval for the percentage of students in the population who would be classified as obese.
89. The offspring of a cross between fruit flies of two certain genotypes yields offspring of two types, those with white eyes and those with red eyes. A geneticist experimentally crosses those two genotypes and counts the two kinds of offspring produced. She finds that there are 17 white eyed offspring and 56 red eyed offspring. (a) Give a 95% confidence interval for the true proportion of white eyed offspring from this type of cross. (b) Interpret the confidence interval.
90. A coin is tossed 1000 times so that it lands on a smooth, polished table top. It lands with the heads side up 532 times and lands tails 468 times. (a) Give a 95% confidence interval for the true heads probability of this coin when tossed onto that table top. (b) Does this indicate that the coin is biased toward heads? Explain.
91. A die is tossed 2000 times and the number showing is recorded. In the 2000 tosses, the die shows a one 323 times and shows some other number 1677 times. Give a 95% confidence interval for the true probability this die shows a one on a given toss. (b) Does this indicate that the probability the die shows a one when tossed is different from $1/6$? Explain.
92. A random sample of objects from a certain population consists of 18 type-A objects and 40 type-B objects. (a) Is the sample size large enough for a confidence interval for the proportion of type-A objects in the population to be valid? If so, give a 95% confidence interval for that proportion.

93. In La Crosse Wisconsin, the squirrels come in two colors, grey and Black. During a 5 day interval, 23 squirrels were observed, 5 of them black. Is this a big enough sample that a confidence interval using the z -distribution would be valid? If so, give a 90% confidence interval for the proportion of La Crosse squirrels that are black. If not, say why not.
94. A population of voters is sampled in order to estimate the proportion of people with each of several political affiliations. The sample breaks down as follows:

Affiliation	Democrat	Republican	Independent	Other
# observed	87	62	75	20

- (a) Give a 95% confidence interval for the true proportion of Democrats in the population.
- (b) Give a 95% confidence interval for the proportion of Republicans in the population.
95. A random sample of 60 students at a certain university are asked how much they like country music. The responses are coded “0” for dislike, “1” for like, and “2” for no comment. The data is as follows:

1 0 0 2 1 0 0 0 1 1 2 1
 2 1 0 0 0 1 1 2 1 0 0 0
 0 0 0 1 1 2 1 1 0 2 0 1
 1 2 0 0 1 0 1 0 1 1 0 1
 1 0 0 2 2 1 2 0 1 1 2 1

- (a) Give a 90% confidence interval for the proportion of students at the university who like country music.
- (b) Give a 90% confidence interval for the proportion of students who would not comment.
96. A population consists of two types of objects, type A and type B . A magazine article states that $1/4$ of the objects in the population are of type A . A random sample from the population consists of 122 of type A and 230 of type B . (a) Give a 95% confidence for the proportion of type A objects in the population. (b) Give a 95% confidence interval for the proportion of type B objects in the population. (c) Can we conclude with 95% confidence that the statement in the magazine article is incorrect?
97. Suppose the sequence of letters in this sentence may be considered to be a random sample from the sequence of all letters that appear in this entire problem set. Using that assumption, construct a 95% confidence interval for the relative frequency of the letter “e” in this problem set.
98. A random sample from a normally distributed population consists of the values, 7, 4, 8, 6, 9, 11. (a) Give a 95% confidence interval for the population mean. (b) How long is the confidence interval in part (a)? (c) What sample size would be appropriate for estimating the population mean to within ± 0.4 with 95% confidence?

99. In a pilot study of a certain population, a variable, K , is being investigated. The study shows that the mean and standard deviation of K are approximately 47 and 3 respectively. What sample size would be required to estimate the mean value of K for the population to within ± 0.5 with 90% confidence?
100. A random sample from a normally distributed population consists of the values, 17, 19, 17, 22, 25. (a) Give a 95% confidence interval for the population mean. (b) What is the length of that interval? (c) What sample size should we use if we want to produce a 95% confidence interval of length 2 for the population mean?
101. A random sample from a population of first grade school-children consists of 41 boys and 67 girls. (a) Give a 90% confidence interval for the proportion of boys in the population. (b) What sample size would be required to estimate the true proportion of boys in the population to within plus or minus .05, with 90% confidence?
102. It is known that in a certain population of measurements the maximum value is 153 while the minimum value is 86. What sample size would be appropriate for estimating the population mean to within plus or minus 2 with 95% confidence? (The idea here is to use your intuition about the empirical rule to make a rough guess at how many standard deviations would roughly equal the range. Then this leads to a rough guess at the standard deviation.)
103. What sample size from a population of adults would be needed to produce a 99% confidence interval of length not over .09 for the proportion of adults who prefer Coke over Pepsi?
104. To produce a 95% confidence interval of length 3 for the mean of a population for which a pilot study suggests that the mean and standard deviation are 52.3 and 7.8 respectively, what size sample would be appropriate?
105. During rush hour on I-90 near La Crosse, Wisconsin, a random sample of 10 cars were clocked at various speeds. The mean was 68 mph with standard deviation 7 mph. (a) give a 90% confidence interval for the mean speed of cars on I-90 during rush hour. (b) What sample size would be needed to estimate the mean speed of those cars to within ± 1 mph with 95% confidence?
106. A certain loaded die lands with the 6 side up a little over $1/6$ of the time. The die is tossed 150 times. In these 150 tosses a '6' was obtained 30 times. (a) Give the relative frequency with which the outcome, '6' occurred. (b) Give a 95% confidence interval for the probability this particular die will land with the '6' side up. (c) How many tosses would be required to determine the "6" probability to within plus or minus .02 with 95% confidence?
107. A random sample from a normally distributed population consists of the data values, 47, 49, and 42. (a) Give a 99% confidence interval for the population mean. (b) Estimate the sample size needed to determine the true population mean to within plus or minus 2.5 with 99% confidence.
108. A physicist wishes to measure a certain physical constant to within $\pm 5.0 \times 10^{-6}$ with 95% confidence. The standard deviation of the measurements are expected to be

approximately 2×10^{-5} . If this turns out to be correct, what sample size would be needed to achieve the desired level of precision?

109. A certain state governor was well liked two years ago, but she has become quite unpopular recently. Now it is desired to estimate her approval rating among voters throughout the state to within $\pm 4\%$ with 95% confidence. It is believed that her approval rating is under 15%. (a) Assuming this belief is correct, how many voters should be polled in order to determine the governor's true approval rating to the specified accuracy and confidence? (b) Without assuming anything a priori about the approval rating, how many voters would need to be polled to be certain of achieving the specified accuracy with the specified confidence?
110. computer exercise The data file 'BROIQ.TXT' contains three columns of numbers. The first column is an ID number. The second column is the IQ measurement of an adult male and the third column contains the IQ measurement of an adult male who is a younger brother of the adult male in the second column. The pairs of brothers were selected at random from pairs of brothers living in the city of Chicago. Give a 95% confidence interval for the mean IQ of the population of older brothers and give one also for the population of younger brothers. State any assumptions required for the validity of the intervals.
111. A chemist has a dilute water solution of a certain acid and wishes to determine the exact concentration. She will do this by titrating the solution with a basic solution of known concentration. She performs the experiment four times, accumulating the following four experimental values (in moles per liter).

$$2.14 \times 10^{-3}, \quad 2.49 \times 10^{-3}, \quad 2.33 \times 10^{-3}, \quad 2.23 \times 10^{-3}.$$

Give a 95% confidence interval for the true concentration of the solution, and state any assumptions required for the validity of your result.

112. Three measurements are made of a certain physical constant, K . The values obtained are .00021, .00025, and .00026. Give a 95% confidence interval for this constant.