Summing Amplifier

The **Summing Amplifier** is a very flexible circuit based upon the standard *Inverting Operational Amplifier* configuration. In the Inverting Amplifier if we add another input resistor equal in value to the original input resistor, $R_{in}$ we end up with another operational amplifier circuit called a **Summing Amplifier**, "Summing Inverter" or even a "Voltage Adder" circuit as shown below.

The output voltage, $(V_{out})$ becomes proportional to the sum of the input voltages, $V_1$, $V_2$, $V_3$,..etc. So, the original equation for the inverting amplifier can be modified to take account of these new inputs thus:

$$I_F = I_1 + I_2 + I_3 = -\left[ \frac{V_1}{R_{in}} + \frac{V_2}{R_{in}} + \frac{V_3}{R_{in}} \right]$$

$$V_{out} = -\left[ \frac{R_F}{R_{in}} V_1 + \frac{R_F}{R_{in}} V_2 + \frac{R_F}{R_{in}} V_3 \right]$$
Summing Amplifier

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\[ V_{out} = -\left[ \frac{R_F}{R_{in}} V_1 + \frac{R_F}{R_{in}} V_2 + \frac{R_F}{R_{in}} V_3 \right] \]

- However, if all the input impedances, \((R_{in})\) are equal in value the final equation for the output voltage is given as:

\[ V_{out} = -\left( \frac{R_F}{R_{IN}} \right) (V_1 + V_2 + V_3 \ldots etc) \]

A **Scaling Summing Amplifier** can be made if the individual input resistors are "NOT" equal. Then the equation would have to be modified to:

\[ V_{out} = -V_1 \left( \frac{R_F}{R_1} \right) + V_2 \left( \frac{R_F}{R_2} \right) + V_3 \left( \frac{R_F}{R_3} \right) \ldots etc \]

The **Summing Amplifier** is a very flexible circuit indeed, enabling to effectively "Add" several individual input signals. If the inputs resistors, \(R_1, R_2, R_3\) etc, are all equal a unity gain inverting adder can be made. However, if the input resistors are of different values a "scaling summing amplifier" is produced which gives a weighted sum of the input signals.
Summing Amplifier: Example 1

- Find the output voltage of the following *Summing Amplifier* circuit.

Solution:
Using the previously found formula for the gain of the circuit

\[ \text{Gain} = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}} \]

substitute the values of the resistors in the circuit to have,

\[ V_{out} = -10(2\text{mV}) - 5(5\text{mV}) = -45\text{mV} \]
Differential Amplifier

By connecting one voltage signal to one input terminal and another voltage signal to the other input terminal the resultant output voltage will be proportional to the "Difference" between the two input signals of V1 and V2 and this type of circuit can be used as a Subtractor and commonly known as a Differential Amplifier, with configuration and as shown below:

The transfer function for a Differential Amplifier circuit is given as:

$$V_{OUT} = -\frac{R_2}{R_1}V_1 + \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)V_2$$

When $R_1 = R_3$ and $R_2 = R_4$ the transfer function formula can be modified to the following:

$$V_{OUT} = \frac{R_2}{R_1} (V_2 - V_1)$$

If all the resistors are all of the same ohmic value the circuit will become a Unity Gain Differential Amplifier and the gain of the amplifier will be 1 or Unity.
The **Differential Amplifier** is a very useful op-amp circuit and by adding more resistors in parallel with the input resistors $R_1$ and $R_3$, the resultant circuit can be made to either "Add" or "Subtract" the voltages applied to their respective inputs. One of the most common ways to achieve this, is to connect a "Resistive Bridge" commonly called a **Wheatstone Bridge** to the input of the amplifier as shown below.

The Differential Amplifier circuit becomes a differential **voltage comparator** by "Comparing" one input voltage to the other. For example, by connecting one input to a fixed voltage reference and the other to either a "Thermistor" or a "Light Dependant Resistor" the amplifier circuit can be used to detect either low or high levels of temperature or light as the output voltage becomes a linear function of the changes in the active leg of the resistive bridge and this is shown below.
Differential Amplifier: application

• Light Activated Switch

This circuit acts as a light-activated switch, which turns the output relay either "ON" or "OFF" as the light level detected by the LDR resistor exceeds or falls below the pre-set value of VR1. The fixed voltage reference is applied to the inverting input terminal V1 via the R1 - R2 voltage divider network and the variable voltage (proportional to the light level) applied to the non-inverting input terminal V2. It is also possible to detect temperature using this type of circuit by simply replacing the Light Dependant Resistor (LDR) with a thermistor.
The Integrator Amplifier

- If the purely Resistive ($R_f$) feedback element of an inverting amplifier is changed to a Frequency dependant Impedance (such as a Capacitor, $C$), an RC Network across the operational amplifier known as integrator circuit is resulted as shown below.

The Integrator Amplifier is an operational amplifier circuit that performs the mathematical operation of Integration.

\[
\frac{V_{in}}{R} = -C \frac{dV_{out}}{dt}
\]

\[
V_{out} = -\frac{1}{RC} \int V_{in} \, dt
\]
The Integrator Amplifier (AC)

- If the input signal is of a sine wave of varying frequency the **Integrator Amplifier** begins to behave like an active "**Low Pass Filter**", passing low frequency signals while attenuating the high frequencies. However, at DC (0Hz) the capacitor acts like an open circuit blocking any feedback voltage resulting in zero negative feedback from the output back to the input of the amplifier. Then the amplifier effectively is connected as a normal open-loop amplifier with very high open-loop gain resulting in the output voltage saturating.

- The addition of a large value resistor, R2 across the capacitor, C gives the circuit the characteristics of an inverting amplifier with finite closed-loop gain of Rf/Rin at very low frequencies while acting as an integrator at higher frequencies. This then forms the basis of an **Active Low Pass Filter**
The Differentiator Amplifier

- The basic **Differentiator Amplifier** circuit is exact the opposite to that of the **Integrator** operational amplifier circuit. The position of the capacitor and resistor have been reversed.
- This circuit performs the mathematical operation of **Differentiation**, so it produces a voltage output which is proportional to the input voltage's **rate-of-change** and the current flowing through the capacitor.

\[
i_{\text{IN}} = I_F \quad \text{and} \quad I_F = -\frac{V_{\text{OUT}}}{R_F}
\]

\[
i_{\text{IN}} = C \frac{dV_{\text{IN}}}{dt} = I_F
\]

\[
\therefore \quad -\frac{V_{\text{OUT}}}{R_F} = C \frac{dV_{\text{IN}}}{dt}
\]

\[
V_{\text{OUT}} = -R_F C \frac{dV_{\text{IN}}}{dt}
\]
The Differentiator Waveform

- If we apply a constantly changing signal such as a Square-wave, Triangular or Sine-wave type signal to the input of a differentiator amplifier circuit the resultant output signal will be changed and whose final shape is dependant upon the RC time constant of the Resistor/Capacitor combination.
Opamp: Problems

1. Use nodal analysis and virtual short and open circuit principle to find output voltage \( v \) and current \( i \).
The circuit of Fig. 4.27 has two unknown node voltages, labeled $v_1$ and $v_2$. By the virtual short principle, the inverting input terminal is at ground potential, and we have labeled its node voltage 0 V. Thus we need two node equations. For circuits not containing op amps, we write KCL at each node or supernode containing an unknown node voltage. Where op amps are present, we modify that strategy by writing KCL at the inverting input node rather than the op amp output node. At nodes 1 and the noninverting input node, respectively,

$$
\frac{1}{10 \times 10^3} + \frac{1}{20 \times 10^3} v_1 + \frac{1}{10 \times 10^3} (v_1 - 5) = 0 \quad (4.27a)
$$

$$
\frac{1}{10 \times 10^3} (0 - v_1) + \frac{1}{5 \times 10^3} (0 - 5) + \frac{1}{5 \times 10^3} (0 - v_2) = 0 \quad (4.27b)
$$

The virtual open principle was used in (4.27b). Solving (4.27a) for $v_1$ yields

$$
v_1 = 2 \text{ V}
$$

and substituting into (4.27b) gives

$$
v_2 = \frac{1.2}{-0.2} = -6 \text{ V}
$$

Then the desired unknowns are $i = -6/1000 = -6 \text{ mA}$ and $v = 5 - v_1 = 3 \text{ V}$. 
2. Use nodal analysis and virtual short and open circuit principle to find the ratio \(v_2/v_1\) \((\text{ans} \ -4/5)\)
Suppose that at $t = 0$, the output in Fig. 6.10 is $v_2(0) = V_0$ and that $v_g(t) = 0$ for $t > 0$. We wish to find $v_2(t)$ for $t > 0$. By the virtual short principle, node 1 is at ground potential. Summing currents into this node gives
\[ \frac{v_g - 0}{R_A} + \frac{v_2 - 0}{R_F} + C \frac{d(v_2 - 0)}{dt} = 0 \]

or, with $v_g = 0$ for $t > 0$,
\[ \frac{dv_2}{dt} + \frac{1}{R_F C} v_2 = 0 \]

This is our standard unforced first-order differential equation with the solution
\[ v_2(t) = v_2(0)e^{-t/R_F C} = V_0 e^{-t/R_F C} \]
3-The circuit is in steady state before the switch is opened. Find the voltage \( v_2 \) after the switch is opened (\( t > 0 \))
With the op amp input shorted to ground for \( t < 0 \), \( v_1 \) and \( v_2 \) are zero for \( t < 0 \). For \( t > 0 \), the voltage at node 1 is 10 V, since the voltage at the noninverting input is 10 V by the virtual open principle, and \( v_1 \) equals this value by the virtual short principle. The node equation at the inverting input node is then

\[
\left( \frac{1}{25 \times 10^3} \right) (10 - 0) + (50 \times 10^{-6}) \frac{d}{dt} (10 - v_2) \\
+ \left( \frac{1}{20 \times 10^3} \right) (10 - v_2) = 0
\]

or

\[
\frac{dv_2}{dt} + v_2 = 18 \quad (6.22)
\]

The trial forced solution is \( v_{2f} = A \), and plugging it into (6.22), \( A = 18 \). The characteristic equation of the unforced version of (6.22) is \( s + 1 = 0 \), or

\[
v_{2n}(t) = Ke^{-t}
\]
Then the total solution is
\[ v_2(t) = v_{2n}(t) + v_{2f}(t) = Ke^{-t} + 18 \]

At \( t = 0^+ \), there is no current flow through the 10-kΩ resistor, so the voltage at the noninverting input is 10 V. Since no voltage drop occurs across the op amp input terminals (the virtual short principle), \( v_1(0^+) = 10 \text{ V} \) also. Continuity of the capacitive voltage requires that
\[ v_2(0^+) - v_1(0^+) = v_2(0^-) - v_1(0^-) = 0 - 0 = 0 \]

Thus \( v_2(0^+) = 10 \text{ V} \) also, and evaluating the constant \( K \), we obtain
\[ v_2(0^+) = K(1) + 18 = 10 \text{ V} \]
or \( K = -8 \). Thus
\[ v_2(t) = 18 - 8e^{-t} \text{ V}, \quad t \geq 0 \quad (6.23) \]

which, combined with the earlier observation that \( v_2(t) = 0 \) for \( t < 0 \), completes the solution for all \( t \).
Find the describing equation for $v_2$ and the unit step response.

**Answer** $d^2v_2/dt^2 + 2(dv_2/dt) + v_2 = 0; \quad v_2 = -te^{-t}$